Experimental and Numerical Analysis of Seismic Response of Unreinforced Masonry Cross Vaults

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Abstract.

The present paper shows an experimental and numerical analysis to understand the seismic behaviour of unreinforced masonry cross vault. The experimental tests were performed on a 1:5 scale model of a cross vault made of 3D-printed blocks with dry joints. The seismic actions was experimentally simulated as a horizontal force proportional to the vault's mass by using a quasi-static tilt testing setup. The vault 3D collapse mechanism and its strength expressed in terms of collapse multiplier was investigated, also considering the direction of the seismic action with respect to the vault's base. The tests results were compared to those obtained from a numerical analysis using a rigid-block model based on 3D limit analysis. The model formulation allows to take into account both associative and non-associative behaviour. A sensitivity analysis on friction angle variation was also investigated to evaluate the accuracy and robustness of the model.

Keywords: Masonry cross vaults, 3D collapse mechanisms, 3D-printed small scale models, rigid block limit analysis, point-based contact model.

1. INTRODUCTION

The growing cultural awareness of preserving our historical unreinforced masonry (URM) buildings in addition to their severe vulnerability to seismic actions have been lead to the development of computational methods of analysis. However, their adoption to analyse vaulted structures is still undeveloped because of the complexity in understanding their three-dimensional behaviour. In published literature, many studies on different modelling strategies for URM vaults were done. However, only few of them are specifically addressed to their seismic analysis [Milani *et al.* 2014, 2016, Gaetani *et al.* 2017]. A further effective tool of analysis consists in experimental investigations that aim to better understand the three-dimensional behaviour of URM vaults and to provide suitable data to verify the reliability of theoretical models to simulate their response. Recently, several authors have been proposed this experimental approach for analysing both 2D [Calderini and Lagomarsino 2014; Calderini *et al.* 2014] and 3D masonry structures made of discrete units [Zessin *et al.* 2010, Quinonez *et al.* 2010; Van Mele *et al.* 2012; Shapiro 2012]. The main reasons are that (1) the structural response of masonry structures depends more on stability rather than strength [Heyman 1995], (2) the testing setup is less expensive than that of full-scale models, and (3) tests may be repeated after the collapse. Only few authors dealt with the seismic analysis of vaulted structures [Milani *et al.* 2016, Rossi *et al.* 2016]. An extensive literature review and classification of experimental, analytical, and numerical analysis is presented in Rossi [2015].

This paper analyses the seismic response of an unreinforced masonry cross vault, which were some of the most common floors of historical buildings. The analysis was performed experimentally by testing a 1:5 scale model made of 3D-printed discrete blocks assembled with dry joints. Seismic action was simulated as a horizontal force using a tilting table. The force is proportional to the gravitational load that, in this case, was only the vault's mass. Six testing cases were analysed aiming to evaluate the response sensitivity to the seismic action direction. In each case, the vault's base was fixed to the table with a different rotation with respect to the axis perpendicular to the table. In particular, the model was rotated by 9° in a range between 0° and 45°. In addition to the experimental analysis, a numerical limit analysis of the vault subject to horizontal loads was performed using a rigid block model developed by some of the authors. The main purposes of the paper are to study the seismic behaviour of URM cross vaults in terms of (1) 3D damage mechanisms and (2) collapse multipliers, and (3) to verify the reliability of the rigid block model to analyse the behaviour of cross vaults.

2. EXPERIMENTAL TESTS

The experiments were performed at the Laboratory of structural enigineering of the Department of Civil, Chemical and Environmental Engineering (DICCA) at the University of Genoa. Section 2.1 illustrates the building of the physical model, from the design of the digital model to the production of the 3D-printed model. Section 2.2 describes the testing setup and the tests results.

2.1 PHYSICAL MODEL

The experimental tests were performed on 1:5 scale model of a cross vault made of discrete blocks. The vault model represent a URM brick cross vault with a square base with a net span of approximately 3.1 m. The model was made up of discrete blocks that had standard dimensions of bricks (0.06 x 0.12 x 0.24 m^3). The total amount of the blocks is 1032. The span l of the scale model was 0.620 m, the rise r 0.225 m, and the thickness t 0.024 m. The masonry patterns and steretomy of the blocks at the intersections of the webs were carefully defined by observing real vaults and by studying previous works [Cangi 2005]. To compensate geometrically the lack of mortar joints, the webs blocks (colored blocks in Figure 2.1) were slightly trapezoidal. The diagonals blocks (light grey blocks in Figure 2.1 and Figure 2.2a) had a special steretomy to guarantee interlocking between the webs. To facilitate the assembling of the vault, each block was labelled (Figure 2.2a). The entire model rested on four rigid plastic abutments (Figure 2.3).

The blocks were made by a 3D prototyping technique called SLS (Selective Laser Sintering). Based on plastic powder sintering, this method proved its effectiveness in building small-scale models with a high geometrical tolerance starting from a 3D digital model. Moreover, the used material had a high hardness to minimize damage due to impacts and to guarantee the repeatability of the tests. The value of friction coefficient μ was determined by testing 12 couples of blocks, giving a value of about 0.56. The elastic modulus E was around 120 MPa and it was calculated by testing in simple compression three assemblies, each ones made up of six blocks. The density ρ of the plastic material was around 0.55 g/cm^3. Since this quite low value would compromised model stability under accidental actions, the weight of the model was increased by inserting a steel plate into each block (Figure 2.2b). Finally, the density of the equivalent material of the blocks (plastic and steel) was of about 2.70 g/cm^3 and the total mass of the model was around 35.6 kg.

To build the model, a plywood scaffolding made of four pieces (Figure 2.4a), corresponding to the vault webs was designed. The scaffolding could be removed making each of the pieces slide on inclined aluminium rails (Figure 2.4b). The global view of the vault's model is shown in Figure 2.5.

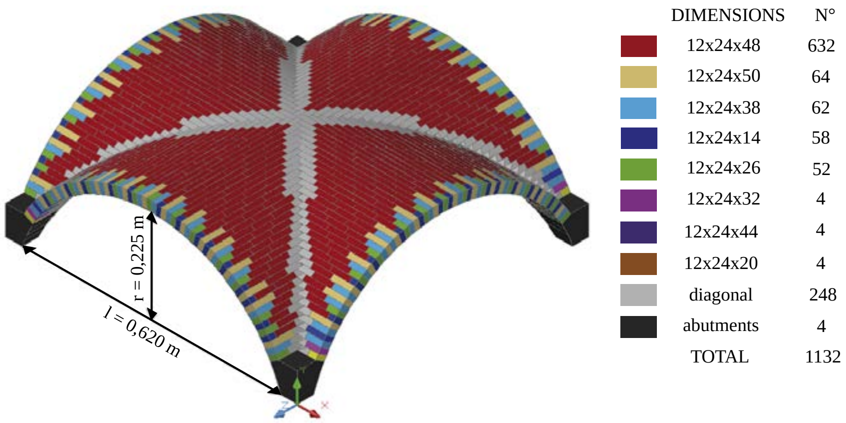


Figure 2.1. Geometry of the vault model.

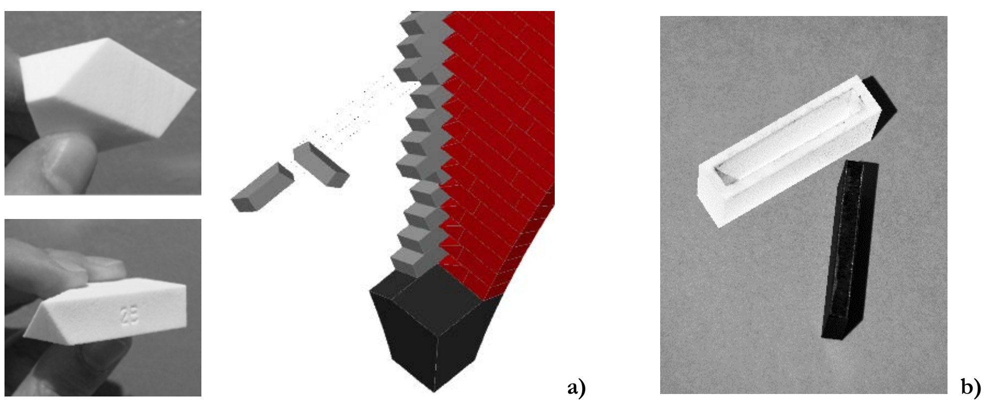


Figure 2.2. Stereotomy of the diagonal blocks (a), and view of a single block with the inserted steel plate (b).

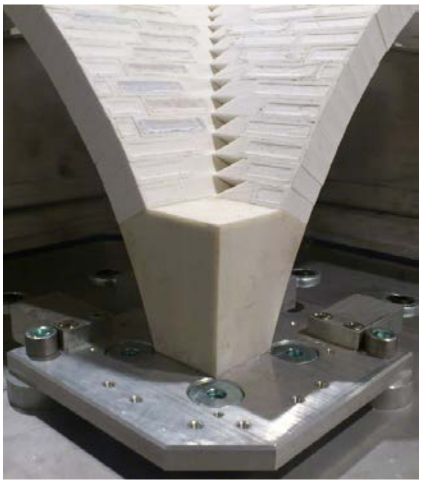
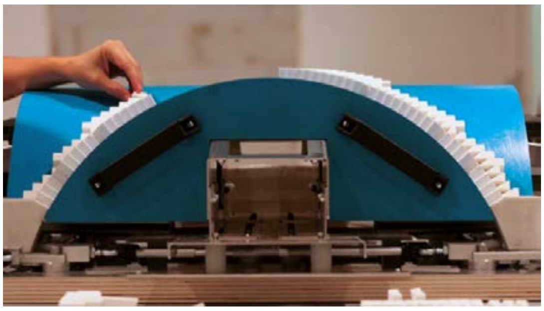


Figure 2.3. Interlocking between blocks at the intersection of the webs and vault's abutments.


a)

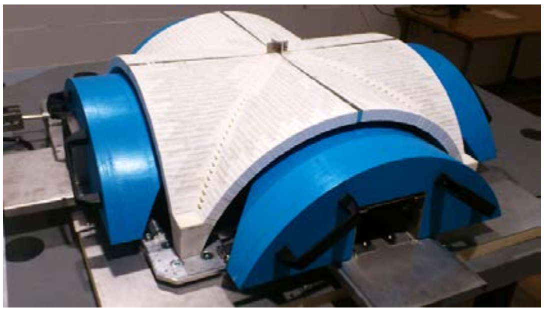

b)

Figure 2.4. Assembly of the blocks on the plywood scaffolding (a), and removing of the pieces by the inclined rails.

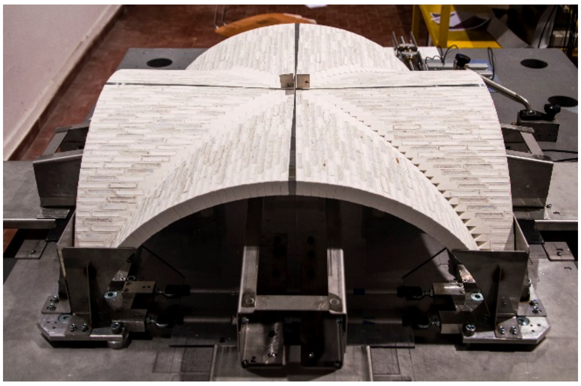


Figure 2.5. View of the built vault's model.

2.2 TESTING SET-UP AND RESULTS

To perform the tests, a tilting table was designed on purpose and the models was rigidly fixed to it. The table was progressively inclined of a certain angle α (Figure 2.6a) until the collapse. The angle α at collapse corresponds to the collapse multiplier that is proportional to the gravitational load of the vault's model. Six testing cases were analysed aiming to evaluate the response sensitivity to the seismic action direction. In each case, the vault's base was fixed to the table with a different angle ϕ with respect to the axis perpendicular to the table. In particular, the model was rotated by 9° in a range between 0° and 45° (Figure 2.6b).

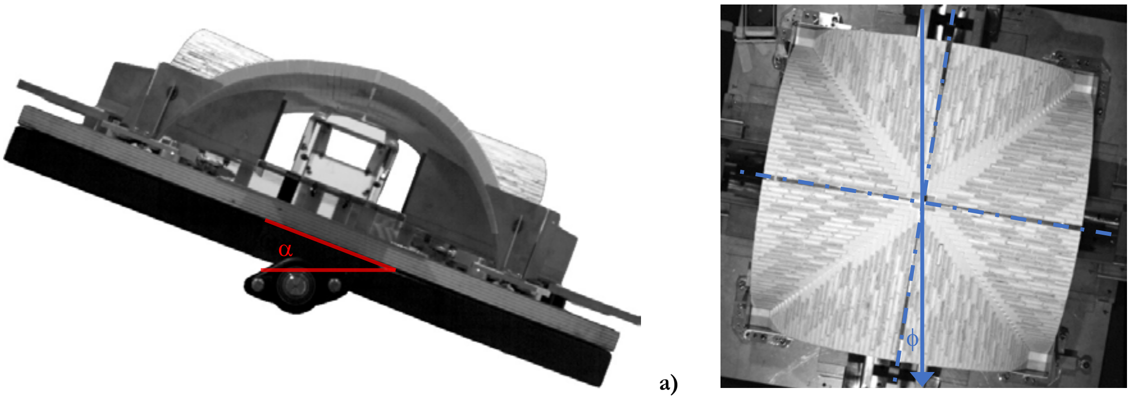


Figure 2.6. Angle of rotation α of the tilting table (a), and angle ϕ with respect to the axis perpendicular to the tilting table (b).

During the tests, α was measured by means of an inclinometer up to collapse. Moreover, two high-resolution/high-frame cameras placed at the top and the lateral view of the model recorded the development of the collapse mechanisms. The results are shown in terms of both damage mechanisms (Figure 2.7 and Figure 2.8) and collapse multipliers versus angle ϕ (Figure 2.9). Figure 2.7 shows the typical collapse modes of the vault according to three angles of rotation ϕ . It can be observed that, at ϕ equal to 9° (as well as at ϕ equal to 0° that is not showed in the figure), the two webs that have the directrix parallel to the rotation axis, showed the development of symmetrical four-hinge mechanism and of a diagonal cracking on the extrados. The collapse mechanism was symmetric with respect to the longitudinal axes of symmetry of the vault. At ϕ equal to 45° (as well as at ϕ equal to 36°), the four-hinges mechanism was symmetric with respect to its diagonal axes of symmetry. The symmetry was lost by performing the tests at ϕ equal to 18° and 27° because of torsional effects that caused an asymmetrical cracking pattern. Figure 2.8 shows the development of collapse mechanism for ϕ equal to 9°. Despite the value of friction would have avoided sliding between blocks surfaces, some sliding phenomena happened.

Figure 2.9 shows the vault's resistance domain as a function of the direction of the seismic action ϕ. The graph shows that value of the collapse multiplier is almost constant in the range 18°÷19.2°. Only in the test with ϕ equal to 18°, the collapse occurred at the lower value of 16.5°, probably because of imperfections caused by the assembly of the blocks model. The maximum strength capacity was achieved at ϕ qual to 0° and to 45°, corresponding to the seismic action along the two axis of symmetry, longitudinal and diagonal, respectively.

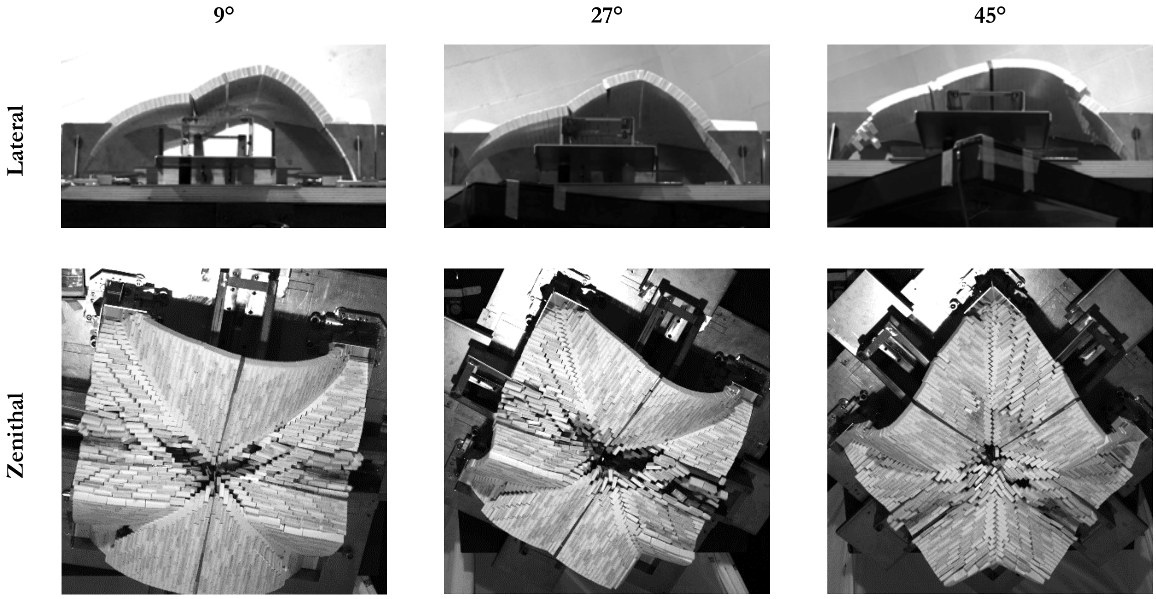


Figure 2.7. Zenithal and lateral view of the vault at collapse for different values of the angle ϕ.

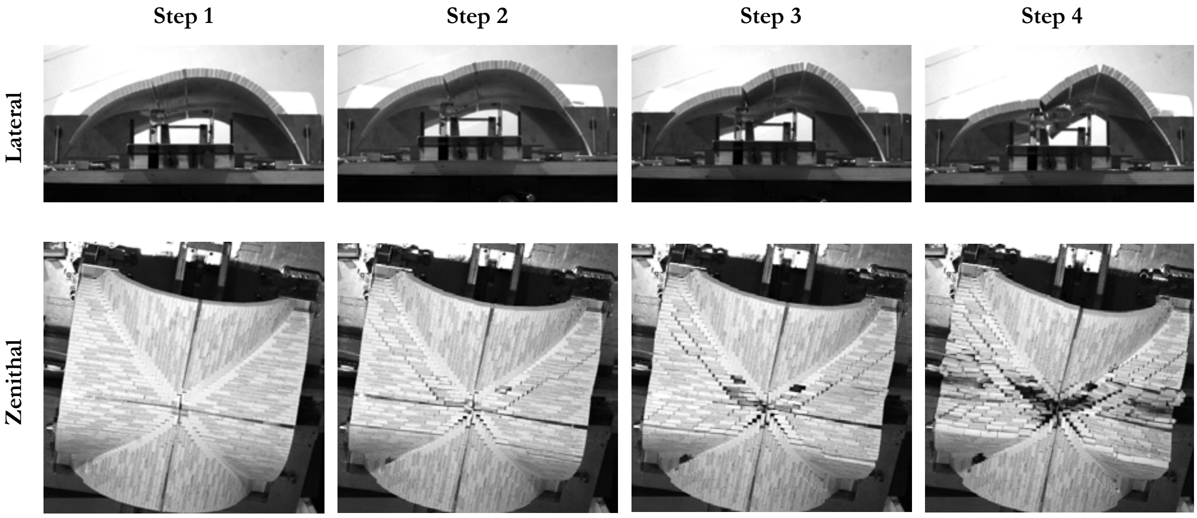


Figure 2.8. Damage evolution of the vault loaded at ϕ = 9°.

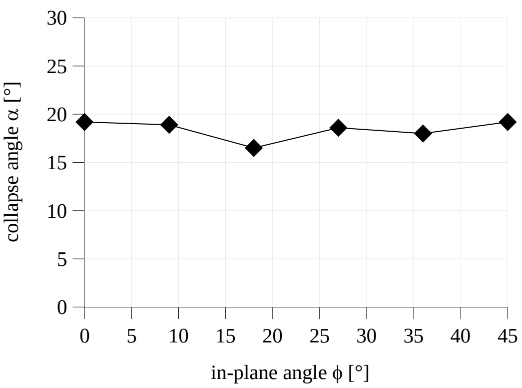


Figure 2.9. Resistance domain of the vault as function of the direction of the seismic action: experimental results.

3. NUMERICAL ANALYSIS

A rigid block model was implemented for limit analysis of the vault under variable lateral loads.

The rigid block model of the vault is shown in Figure 3.1 and comprises 1128 blocks. The numerical model was generated from a CAD solid model imported in the Matlab environment. In the CAD model 87 block typologies were defined to take into account different geometry and interfaces varying with the position of the block in the vault. In particular, 16 typologies were defined to reproduce the frontal arches, 4 block types for the shells, and 67 for the ribs.

3.1 THE RIGID BLOCK MODEL

The numerical model is composed of rigid blocks *i* interacting at contact points *k* located at the vertexes of the interface *j* [Portioli *et al.* 2014, 2016a, 2016b, 2017]. An interface is defined as the overlapping area between two adjacent blocks. Four vertexes are defined for each interface and these represent the contact points where the static variables are applied (Figure 3.2). The model that we adopted for contact interaction is a point-based contact model. This model represents a simple alternative to the classic surface contact model (with static variables associated to stress resultants acting at a single point, i.e. the center of contact interface) which allows to dramatically simplify the formulation of the mathematical programming problems associated to limit analysis.

The static variables are collected in vector $\boldsymbol{c}$ and include the shear force componenta t_{1k} and t_{2k} and the normal force n_k along the local coordinate axes. External loads applied to the centroid of rigid block *i* are collected in vector of external forces $\boldsymbol{f}$. External loads are expressed as the sum of dead and live loads multiplied by the collapse load factor α, as follows:

$$\boldsymbol{f} = \boldsymbol{f_D} + \alpha \boldsymbol{f_L} \tag{1}$$

The corresponding kinematic variables are the relative tangential and normal displacement rates at the contact points.

Equilibrium conditions for the whole rigid block assemblage are expressed in matrix form as follows:

$$\boldsymbol{A}\,\boldsymbol{c} = \boldsymbol{f} \tag{2}$$

For failure conditions, a no-tension frictional behaviour with infinite compressive strength is assumed at contact interfaces.

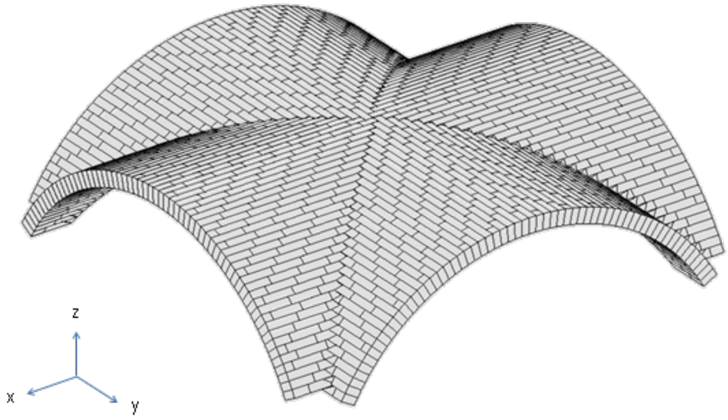


Figure 3.1. The rigid block model of the vault.

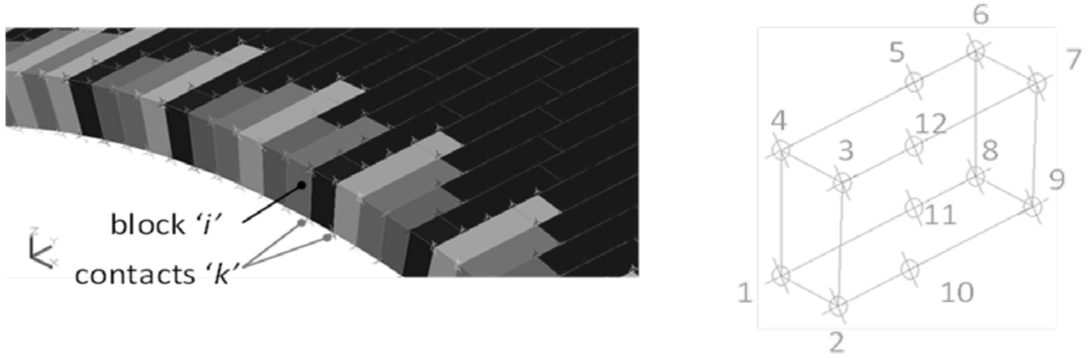


Figure 3.2. Block 'i', contact points and contact interfaces ([1,4,5,11]; [11,5,6,8]; [6,7,8,9]; [7,9,10,12]; [2,3,10,12]).

3.2 FORMULATION OF THE LIMIT ANALYSIS PROBLEM

The calculation of the collapse load multiplier is obtained from the solution of the static formulation of the limit analysis problem. Under the assumption of associative (i.e. dilatant) flow rule, the limit analysis problem can be expressed in terms of the following second order cone programming.

$$\begin{aligned} \max \quad & \alpha \\ \text{s.t.} \quad & \boldsymbol{A}\,\boldsymbol{c} = \boldsymbol{f_D} + \alpha \boldsymbol{f_L} \\ & \boldsymbol{c} \in C \end{aligned} \tag{3}$$

where the second condition represent the Coulomb failure condition described by the convex cones:

$$C = \left\{ \mu\, n_k \geq \sqrt{t_{1k}^2 + t_{2k}^2}\,,\; n_k \geq 0 \right\}, \tag{4}$$

being μ the friction coefficient.

A simple iterative solution procedure of programs (3) was implemented to take into account non-associative (non-dilatant) behaviour for sliding failure, for which a lower and hence safe value of the collapse load multiplier can be obtained, if compared to the associative solution (involving dilatancy).

To take into account non-associative sliding behaviour, iterations are carried out using a fictitious failure condition to restore the normality flow rule for sliding failure with non-dilatant behaviour.

More in detail, the iterative procedure is based on the assumption that at each step of the analysis sliding behaviour is governed by a fictitious failure condition with an associative behaviour. The fictitious failure condition is characterized by a friction angle which is almost equal to zero and by a fictitious cohesion, which is updated at each iteration. Under the assumption that at each iteration the sliding behaviour follows the associative flow rule, the programming problem related to the governing equations of the rigid block model can be formulated according to two dual SOCP programming problems, corresponding to the upper and lower bound formulations of limit analysis [Portioli *et al*, 2014]. The Mosek optimization software was used to solve the mathematical programming problems involved.

3.3 RESULTS OF NUMERICAL ANALYSIS AND SENSITIVITY TO FRICTION COEFFICIENT

For the numerical analysis of the vault subjected to testing on the tilting table, the value of the friction coefficient was set in the range 0.50-0.60 and the unit weight is 27e-6 N/mm^3.

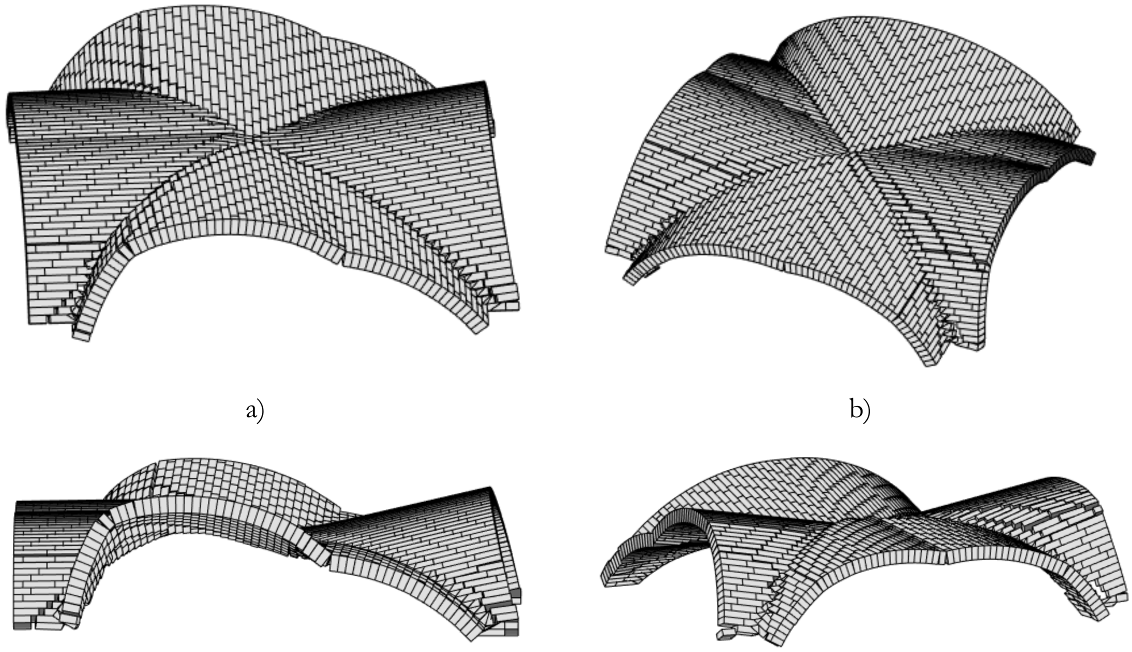
To reproduce the loading condition on the tilting table, the blocks are loaded under dead loads equal to the self weight and variable horizontal loads expressed as the dead load multiplied by the load factor α.

Sensitivity analysis to friction coefficient and loading direction were carried out to evaluate the accuracy and consistency of the results.

The results are summarized in the following Table.

Table 1. Sensitivity analysis of collapse load multiplier to friction angle and loading direction

Loading axis	Friction angle	Associative	Non-associative solution
		α_{ass}	α
	0.50	0.41862	0.33997
+ X	0.56	0.52353	0.40571
	0.60	0.61205	0.46028
	0.50	0.42772	0.34014
- X	0.56	0.52236	0.40589
	0.60	0.61185	0.46132
	0.50	0.38191	0.30904
+ Y	0.56	0.52046	0.40515
	0.60	0.6106	0.4777
	0.50	0.38236	0.31066
- Y	0.56	0.52159	0.40455
	0.60	0.6085	0.4772



a) b)

c) d)

Figure 3.3. Predicted failure mode.

The comparison of the numerical failure modes with the experimental ones are in good agreement. The numerical failure mode is quite symmetric with respect to the longitudinal axes of symmetry of the vault and a four-hinge mechanism is observed in this direction (Figure 3.3a, and Figure 3.3b). As for the transversal direction, a three hinge mechianism is observed in one web, being the other almost undeformed (Figure 3.3c, and Figure 3.3d). With respect to collapse load multipliers, slight differences are observed between the numerical and experimental results. In particular the associative solution overestimate the experimental one of about 47%. This difference is dramatically reduced to 14% in case of non associative solution caused by the effect of sliding failure on collapse mechanism. For computational efficiency, it is worth noting that the associative solution was computed in 115 seconds.

4.CONCLUSIONS

This paper presents an experimental analysis on a 1:5 scale model of a cross vault made of 3D-printed discrete blocks assembled with dry joints tested by using a tilting table. The seismic behaviour of the vault in terms of both 3D collapse mechanisms and the collapse multipliers is evaluated by simulating the horizontal action along different directions on respect to the lateral view. The tests results are used to verify the reliability of a rigid-block numerical model developed by some of the authors. Concluding considerations are the following.

The main collapse mechanism is a four-hinge mechanism, well observable from the model lateral view, which develops three-dimensionally on the webs surfaces.

The collapse mechanism of all the tests is characterised by the development of a four-hinges mechanism a diagonal cracking that develops on the vault's extrados.

Considering the seismic action direction with ϕ in the range 0° - 9° the four-hinge mechanism and the diagonal cracking are symmetric with respect to the longitudinal axes of symmetry of the vault. On the contrary, testing the vault with an angle ϕ in the range 36° - 45° the symmetry is with respect to the diagonal axes of symmetry.

The tests with ϕ equal to 18° and 27° show a more asymmetrical failure mechanism because of torsional effects prevailing.

The results in terms of collapse multipliers show an independence from the direction of the seismic action. The value lies between 18° to 19.2°.

The maximum strength is achieved at $\phi = 0°$ and $\phi = 45°$, corresponding to seismic actions along the two axes of symmetry, longitudinal and diagonal.

The rigid-block numerical simulation shows a good agreement in terms of mechanisms of collapse.

A remarkable difference was noted between the values of the collapse load multipliers related to the associative behaviour (involving dilatancy in the case of sliding failure) and non-associative solution (no dilatancy). The values of the collapse multipliers obtained using the non-associative friction joints model are closer to the experimental one. This is beacause the non-dilative behaviour in sliding failure involves lower (and hence safe) values of the collapse load multiplier with respect to the associative solution.

The results of this research encourage to future developments. In particular, the flexibility in the use of 3D-printed scale models may allow to perform other mechanisms applying both external loads and displacements. Other geometrical scale and typologies of vaults may be tested. Moreover, the effects of further gravitational loads as the presence of infil may be investigated. The good agreement between experimental tests and numerical simulations encourages to further investigations and developments of the rigid-block model for analysing masonry vaulted structures made of discrete elements.

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